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History of Probability and Its Uses in Everyday Life

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Abstract

A systematic history of the development of probability theory is briefly studied and then its uses in everyday life are discussed. More specifically, after giving a short sketch of the evolution of probability theory, different approaches of defining probability with limitations of each are also presented and afterwards uses and applications of probability theory with examples are cited.

Keywords: Random experiment, Sample space, Random event, Event space, Probability function, Axiom, Theorem, Axiomatic Probability

1. Introduction

Very often we are using the word probability in our everyday life. For example, probability of coming head if we toss a balanced coin or probability of coming an even numbers if we throw a balanced die or probability of drawing an ace if we draw a card from a well shuffled packet of cards or there will be no chance or probability of having rain tomorrow, there is a good chance of winning Bangladesh in the coming ICC tournament, there is a probability of having drought in the coming summer in Bangladesh, there is 5% chance that there will be a third world war within the next two years etc. The events cited above are related with certain degree of uncertainty. These types of events are called random events. Roughly speaking, probability is a numerical measure of the degree of uncertainty of random event.

2. Historical Background

A short historical development of the theory of probability is given in a book written by Roy, M.K.& Roy, D.C. (2021). The term probability and its origin are related with the games of chance popularly known as gambling. Gambling was very popular and fashionable beginning in the last fifteenth century especially in Italy, Greece and French among the higher class of people for recreation. It was not restricted by laws. As the game became more complicated and the stakes became larger, there was a need for mathematical foundation of computing chances.

Historically, the first problem of probability was found in a book of Luca Pacioli (1447-1517), (1494), an Italian mathematician. The name of the book was 'Summa de Arithmetica, Geometrica, Proportioni et Proportionalita published in1494.

First Problem: A and B, playing at a fair game agree to continue until one has won six rounds; but the match has to stop when A has five and B has won three points. How should the stakes be divided?

Solution of Pacioli: The stakes should be divided in the ratio 5:3 as A has won 5 rounds and B has 3 rounds.

The error of the solution of the problem was first noted by Tartaglia (1500-1557), an Italian mathematician in 1556. He argues that the difference between A's score and B's score is 2, and this being one-third of the number of games needed to win. A should take one third of the B share and the total stake should be divided in the ratio 2:1.

This solution was not correct. But the actual solution is 7:1 in favour of A which was proved by French mathematician Blaise Pascal in 1654 after almost one hundred years later the solution of Tartaglia.

Actual Solution by Pascal: Since the game is fair one, the possible outcomes of winning the game are S = {A, BA, BBA, BBB}

The probability of A's winning the game is P[A]+P[BA]+[BBA]=1/2+1/4+1/8=7/8.

The probability of B's winning the game is P[BBB]=1/8.

Hence, the stakes should be divided according to 7:1 in favour A.

Historically, the problem cited above is known as Problem of Points. Initially, games of gambling's were related with coins tossing, dice throwing etc., since playing cards were invented in 1350 only.

Geronimo Cardano (1501-1576): A systematic solution of some problems related to dice were found in a book written by Cardano, a mathematician, Physician, Physicist, Philosopher and Gambler published in 1663 in his book "De Ludo Alae". The manuscript was found in Rome, 1576 after his death. But it was written in 1526. An English version of the book translated by Oystein (1953) named "Carnodano, the Gambling Scholar".

It is notably mentioned the name of another Italian mathematician Galile Galilei (1564-1642), who was related with the game of chances. His book "Sulla Scoperta del dadai" contains a good number of problems related with dice which was published in 1718 after his death although it was written in 1642. In this book, he correctly mentioned that in three dice experiment 10 and 11 are more likely to occur than 9 and 12. In a tabular form, he showed that 10 and 11 come 27 times but 9 and 12 come only 25 times. He was the first man who tried to define a quantitative measure of probability while dealing with the problems of dice in gambling.

It is observed that the mathematical foundation of the games of chances was very slow. Today without any hesitation, we can say that Italy is the land of origin of probability.

Classical Probability

Now we will discuss the role of the mathematician of French for the development of probability. There was a geographical and political connection between French and Italy since fourteenth century. King of French Charles VII attacked Italy in 1494 and captured it for few days.

There was an intellectual connection between the two countries. Some mathematicians settled in French. So, problems of gambling or games of chance were not unknown to the French people. In the seventeenth century, gambling was very popular among the higher classes of people in French. But, the first foundation of the mathematical theory of probability was laid in the mid-seventh century by two French Mathematicians B. Pascal (1623-1662), and P. Fermat (1601-1665), while solving a number of problems posed by French nobleman and gambler Chevalier-de-Mere to Pascal. His name is connected with the origin of probability. Among the problems which de-Mere posed to Pascal was the "problem of point", which was discussed already.

Another of de-Mere's enquiries involved the throwing of dice. He thought that he had developed a technique for winning gambling with dice. According to Mere, the probability of getting at least one six in throwing a dice once, twice, thrice or fourth times are 1/6, 2/6, 3/6 and 4/6. It is seen that the probability of getting at least one six in throwing a fair die four times is more than half. He won a lot of money by using this strategy. Then, he changed his strategy. According to the same rule, the probability of getting at least double sixes in throwing two dice 24 times should be 24/36. By applying these strategies, he lost all his money. When he lost his bet, he complained to Pascal. Pascal assured him that these results were to be expected. The probability of at least one six in four tossed is $1-(5/6)^4 = 0.518$. But, the probability of at least double six in 24 tosses of a pair of dice is $1-(35/36)^{24} = 0.491$ which is less than half. Motivated by these enquiries, Pascal engaged in a lengthy correspondence with Fermat. This correspondence served as a basis for the unified theory of chance phenomena which we today call the theory of probability.

A letter from Pascal to Fermat on July 29, 1654 contains the following passage: "C.de-Mere told me that he had found a fallacy in the theory of number, for this reason: If one undertakes to get one die, the advantage in getting it in 4 throws is 4/6. If one undertakes to throw double six with two dice, there is disadvantage in undertaking it in 24 throws. And never the 24 to 36 (which is the number of pairings of the faces of two dice) as 4 is to 6 (which is the number of faces of one die). That is what made him so indignant and made him say to one and all that the propositions were not consistent and arithmetic was self-contradictory."

A Dutch mathematician Christiaan Huygens (1629-1695), who had contributed a lot to the development of probability. In 1655, when he was 26 years old, he went to French and came in contact with great French mathematician Robarval. He had a good connection with the mathematician Fermat and Carcavi. He systematically solved a number of problems of points and dice and sent the manuscript to his teacher Van Schootan. Van Schootan translated the manuscript in Latin language in 1657. The same year it was published as a book under the title of “De rationciniis in Alal ludo”. Perhaps, this is the first book on probability. The concept of mathematical expectation is given by him.

Problem of Huygens: A and B throw alternately with a pair of fair dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is 30/61.

Next, Stalwart in this field was J. Bernoulli (1667-1748), who’s “Treatise on probability” was published posthumously by his nephew N. Bernoulli in 1713. Bernoulli trial, Bernoulli distribution, Bernoulli theorem in law of large numbers was some of his important contributions in the field of probability.

Abraham De Moivre (1667-1748), the next French Mathematician who wrote a book on probability. His book “The Doctrine of chances” was first published in 1718. The book was so popular that it was published twice in 1738 and 1754. De Moivre Laplace theorem is one of his important contributions in the field of law of large numbers and it is the first version of Central – Limit Theorem which is used to derive the normal distribution from Binomial.

Thomas Bayes is another important contributor in the field of probability, inverse probability today known as Bayes Theorem which is one of his important contributions.

3. Definitions of Probability

Laplace (1749-1827), was a French astrologer and mathematician. He wrote a number of articles in the field of probability. His book (1956) Foundation of Probability translated from German (Theory “theories Analytique des probabilities” published in 1812) was very popular in Europe. The classical definition of probability in the present is given by him.

3.1 Classical Probability

If there are n equally likely, mutually exclusive and exhaustive outcomes of a random experiment and if m of these outcomes are favourable to an event A, then the probability of the event A is given by

$$P[A] = \frac{m}{n}.$$

An important observation made by the founder of the classical probability – Pascal, Fermat, Huygens, Laplace and J. Bernoulli – was that relative frequency ratio of a given outcome of certain games of chance tends to converge to a definite value when the game is repeated a large number of times. It was, in fact this phenomenon that led them to postulate the classical theory of probability. In games of chance outcomes that can be objectively determined to be equally likely are in fact observed to occur with equal frequency when the game is repeated many times. Thus, according to the classical theory, they are outcomes with equal probabilities. In these ways, classical probability was developed in the seventeenth century with the direct collaboration of mathematician and gamblers.

Initially, the gambling problems related with the games of chances were solved by this classical probability. Even today, classical probability is very important to solve classrooms problem for students. The classical theory can be extended beyond games of chance to any chance situation involving a finite number of equally likely outcomes. The first two rules established by the father of Genetics G. Mendel (1822-1884) were based on classical probability. They are known as Mendel’s First and Second law. In statistical mechanics, how the particles are distributed in the cells of materials under different conditions were proved on the basis of classical probability. They are known as Maxwell-Boltzmann, Bose-Einstein or Fermi-Dirac Statistics. These models are known as “Occupancy problems” in probability theory (1921).

3.2 Empirical or Posteriori or Frequency or Statistical Probability

Classical probability was criticized by the mathematicians like Leibnitz (1646-1716), Von Mises (1883-1953), (1941), Pearson, K (1857-1936), Fisher, R.A. (1890-1962) (1921).

The classical probability has two important limitations:

i. What happens if the outcomes of the experiments are not equally likely which are usually happens in practice.

ii. Moreover, it cannot handle events with an infinite number of possible outcomes
Let us consider the following examples.

A fair coin was tossed 100, 500, 1000 times and the outcomes were recorded in the table 1 and a fair die was tossed 1200 times and the outcomes were recorded in the table 2. The important thing is that the relative frequencies of heads and tails are very close to 1/2. Similarly, the relative frequencies of faces showing points 1, 2, 3, 4, 5 and 6 are very close to 1/6. The results are expected, since the coin and the dice are fair.

Table1. Showing outcomes of tossing a fair coin 100, 500 and 1000 times.

Outcomes	Observed relative frequency			Expected relative frequency
Number of tosses	100	500	1000	
Head	0.46	0.49	0.494	0.50
Tail	0.54	0.51	0.506	0.50

Table2. Showing outcomes of throwing a fair die 1200 times.

Outcomes	Observed Frequency	Observed Relative Frequency	Expected Relative Frequency
1	204	0.1700	0.1667
2	212	0.1767	0.1667
3	192	0.1600	0.1667
4	204	0.1700	0.1667
5	196	0.1633	0.1667
6	192	0.1600	0.1667

From both the experiments, it is reasonable to assume that if we toss a coin or throws a dice, there exists a number say p which will be the probability of head or a face point of the dice. If the coin is fair, this number will be equal to 1/2. In the case of a fair dice, this number will be 1/6.

In the eighteenth and nineteenth century's observers of natural and social sciences found long-run regularity of relative frequency ratios in the natural phenomena relating to demography and experimental data.

One of the first recorded instances was in connection with the sex of new born children at a certain city hospital in England. It was noted that the long-run relative frequency ratio of male births tended towards 1/2. The Outcome of a male birth can be considered a chance phenomenon as is the outcome of head in a toss of coin and that the probability of the outcome was 1/2. In 2238 B. C., it was observed in Chin the relative frequency ratio of male birth was 1/2. To find the male and female birth ratio, Laplace collected birth records from Scant Petersburg, Berlin and whole French and came to the conclusion that this ratio is almost same for all countries. He observed that the male birth ratio is 22/43. Buffin tossed a fair coin 4040 times and Karl Pearson tossed a coin 12 thousand and 24 thousand times and got the number of heads 2080, 6019 and 12012 times respectively and their relative frequencies are 0.5040, 0.5016 and 0.5005 which were very near to 0.5. All these records led to the foundation of definition of probability called empirical or statistical or frequency probability. This theory was developed by Von Mises and R. A. Fisher. Von Mises (1941) empirical definition of probability may be stated as follow**.**

Definition of Empirical Probability: If an experiment is repeated a large number of times under the same conditions, the probability of an event A is the limiting value of the ratio of the number of times that the event A happens to the total number of trials, as the number of trials increases indefinitely large, provided the ratio approaches a finite and unique limit p. That is,

$$P = P\,[A] = \frac{m}{n}$$
$$\text{limit } n \rightarrow \infty$$

where m is the number of times the event A occurs in n trials.

The empirical definition of probability is the most widely accepted definition of probability. It is the definition that most frequently comes to mind when we are confronted with probability statement. For example, when the weather bureau predicts a 0.9 probability of rain, we assume that conclusion to the result of many observations of days with identical weather conditions 90% of which were rain. In other words, 0.9 is an empirical probability of the outcome of rain. The two definitions of probability are apparently different. The empirical probability is the generalization of the classical probability. However, empirical definition of probability is not also free from drawbacks. The following are the drawbacks of this definition.

Drawbacks of the empirical probability

i. In practice, it is not possible to repeat the experiment an infinitive number of times under the identical conditions to get the probability.

ii. Even, if it were possible to repeat an experiment an infinite number of times, it is conceivable that a different infinite sequence of performance of the same experiment could produce a different value for p.

iii. It is not clear how large n should be before we are certain that the probability, p is close to the limiting of $\frac{m}{n}$ as n $\rightarrow \infty$.

The development in the fields of probability was very slow in Europe after Laplace. Russian mathematician made remarkable contributions in the field of probability. Among them, the famous mathematicians are Chebyshev (1821-1894), his student A. Markov (1856-1922), A Lyapuvnov (1858-1918), A H. Khintchine. Chebyshev's inequality in probability theory, Chebyshev's theorem in law of large numbers, Markov's Chain in stochastic process, Lyapuvnov's version of central limit theorem and Khintchine theorem in law of large number are remarkable contributions in the field of probability.

3.3 Axiomatic Probability

This approach was first introduced by the Russian mathematician A. N. Kolmogorov (1903-1987), (1956) in 1933 and now universally accepted by all probability theorists and mathematical statisticians. Perhaps, it is the simplest of all the definitions and least controversial. This definition is based on a number of axioms which allows rigorous development of the mathematics of probability.

Axioms: An axiom is a statement that is assumed to be true.

Theorem: A theorem is a statement that can be deduced either from axioms or from previously proved theorem.

In axiomatic definition of probability, several simple statements concerning probability are assumed to be true. These statements are called the axioms of probability. All other statements concerning probability are deduced from these axioms and they are called the theorems of probability. Such axioms and theorems provide the rules of operation for the mathematics of probability.

Probability functions

Let Ω be the sample space and $\mathcal{A}$ denotes the event space of some random experiment.

Definition: A probability function P [.] is a set function with domain (an algebra of events) and counter domain the interval {0,1} which satisfies the following axioms:

1. P [A] $\geq$ 0 for every A € $\mathcal{A}$

2. P [Ω] = 1

3. Let A_1, and A_2 be mutual exclusive events in $\mathcal{A}$ then

P [A_1 U A_2] = P[A_1] + P[A_2]

4. Let A_1…………., A_n ………… be a sequence mutually exclusive events in $\mathcal{A}$, then

$$P[\cup_1^\infty A_i] = \Sigma_1^\infty P[A_i]$$

The above axioms are termed as the axioms of positiveness, certainty and union, respectively.

The definition of probability is a mathematical definition. It tells us which set function can be called probability function. In our definition of event and $\mathcal{A}$ a collection of events, we stated that cannot always be taken to be collection of all subsets of Ω. The reason for this is that for sufficiently large Ω the collection of all subsets of Ω is so large that it is impossible to define a probability function consistent with the above axioms. We observe that the axioms are indeed reasonable assumptions. Irrespective of classical, empirical and subjective probabilities we would all agree that **-**

A probability is a number between 0 and 1 inclusive. This is axiom 1.

The probability of the sure event is one that that is P [Ω] $=$ 1. This is axiom 2.

If $A_{1,} A_2$ ……. A_n ……. are events that cannot occur simultaneously, the probability that one among them will occur is the sum of these probabilities, this is axiom 4**.**

We may face problems which are not in the frame work of classical or frequency probability. For example, what is the probability that Mr. Hassan will get married this year? Or what is the probability that a third world war will happen in the next 200 years, subjective probability has developed to answer such questions. Answer of all these questions will depend on the individual judgment or belief.

3.4 Subjective Probability

Subjective probability is a personal evaluation of the likelihood of chance phenomena. Subjective probability was developed by Keynes (1921) and Jeffrey's (1961). It is an important element in many decisions making processes and is a basic ingredient for Bayesian decision making processes and is a basic ingredient for Bayesian decision theory. Subjective probability may be defined as follows:

Definition of Subjective probability: The probability that a person assigns to an event which is the possible outcomes of some processes on the basis of his own judgment, beliefs and information about the processes is known as subjective probability.

Likely other definitions, subjective probability of an event is a number, ranging from 0 to 1. As it depends on individual's judgment and belief, it may vary from individual to individual even when they are confronted with the same set of evidence. For example, one fine morning, Mr. Ali may be prepared for rain, but his friend Mr. Ahmed may not.

Subjective probability has the following drawbacks:

i) It varies from individual to individual as it depends on individual's judgment and belief,
ii) It has no objective basis.

3.5 Geometric Probability

From the very beginning of the development of probability theory, an extension of the classical definition of Laplace was used to evaluate the probabilities of sets of events with infinite outcomes. For example, if we are interested in finding the probability that a point selected at random in a given region will lie in a specified part of it, the classical definition is modified and extended to what is called geometric probability. The concept of equal probability of some events played the basic role.

Definition of geometric probability: If Q is some region with well-defined measure, the probability that a point chosen at random lies in a sub region A of Q is called the geometric probability and is defined by the ratio equal to p as

$$p = \frac{measure\ of\ specified\ \ A of\ Q}{measure\ of\ the\, whole\ region\ Q}$$

where measure refers to length, area, volume of the region if we are dealing with one, two or three dimensional space respectively.

Many problems of geometric probability were solved using this extension. The trouble is that one can define at random in any way, one pleases and different definitions therefore lead to different answers. Joseph Bertrand (1889) in his book "cal cul des probabilities" published in 1889 cited a number of problems in geometric probability, where the results depend on the method solution.

Example. The encounter problem: Two persons A and B have agreed to meet at a definite spot between 12 and one O'clock. The first one to come waits for 20 minutes and then leaves. What is the probability of a meeting between A and B if the arrival of each during the indicated hour can occur at random and the times of arrival are independent?

Solution. Denote the time of arrival of A by x and B by y. For the meeting to take place, it is necessary and sufficient that $|x-y|< 20$

We depict x and y as Cartesian coordinates in the plane; for the scale unit we take one minute. All possible outcomes will be described as points of a square with side 60; favourable outcomes will be described as points of a square with side 60; favourable outcomes will lie in the shaded region in fig.3.5.1. The desired probability is equal to the ratio of the area of shaded figure to the area of the whole square.

$$p = \frac{60^2 - 40^2}{60^2} = \frac{5}{9}$$

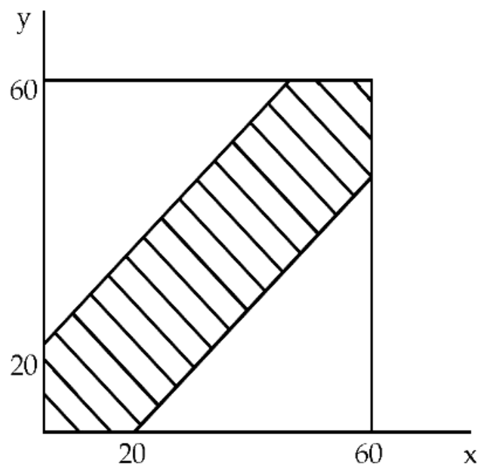

Figure: 3.5.1

4. Uses of Probability

Human life span from birth to death is related with random events. Birth, death, divorce, and accidents are some important random events of human life. Statisticians can successfully find the probabilities of these random events. A man can die at any moment. But, when a person will die is very difficult to say or even impossible except some special cases. But, a statistician successfully constructs a life table of almost every country of the world where the life expectancy and probabilities from birth to death of a community are successfully calculated. Life insurance companies have been doing their business successfully on the basis of this life table. So, we see that the agents of a life insurance companies are running after a young person with a good salary than after a rich person of age 60 or more years.

Random events with probability close to zero or one are very important in probability theory. The laws of large numbers are established for these types of events which play vital role in probability theory. The events are sure whose probabilities are one. These types of events must happen. On the other hand, events with probability zero are impossible events which will never happen. For example, if we toss a coin the event of coming head or tail is a sure event. On the other hand, the event of happening head and tail is an impossible event which will never happen.

It is mentioned here that the probability one does not mean it is a sure event or probability zero does not mean it is an impossible event. For example, the probability that a randomly selected person with height 5.6 foot calculated from the probability distribution of height is zero does not mean that it is an impossible event. The probability of the complementary event is one, does not mean that it is a sure event. Which value of probability is important depends on a particular situation. For example, the distance between Dhaka to Chittagong measured by Mr. Ali says 360 km. If the probability of measuring mistakes up to 5km or more is .02, the distance between Dhaka to Chittagong may be accepted with this error. But for constructing a bridge like Padma bridge or a big power plant like Ruppur Power Plant where requires huge investment in terms of money and manpower, the probability of having flood or natural calamities like earthquake equals to 0.02 is very important. The value of this probability must be taken into account for design in constructing the bridge or the power plant.

The Subject Statistics based on probability theory appeared as an independent scientific discipline of knowledge in the beginning of twentieth century in UK. In 1911 K. Pearson founded the first Statistics Department at university level at University College, London. He is also the founder of the first Statistical Journal "Biometrika" in 1901. Actually, Karl Pearson laid the foundation stone of Statistics as a separate discipline. Some important contributions of Karl Pearson in the field of Statistics are

i. Karl Pearson's Correlation coefficient,
ii. Method of moments
iii. Pearsonian system of Curves
iv. Pearsonian Chi – Square test for goodness of fit
v. Princial component analysis
vi. Neyman – Pearson's lemma in testing hypothesis.

R. A. Fisher is the father of modern Statistics. His main contributions in Statistics are

1. Sampling distribution of t, F and correlation coefficient r
2. Properties of a good estimator
3. Methods of maximum likelihood
4. Fisher' Information
5. Analysis of variance
6. Design of experiments
7. Discriminant analysis

5. Probability Today

Modern research in probability theory is closely related to the mathematical field of measure theory. Modern innovators in the field include "Patrick Bilingsley (University of Chicago), Yuan Shih Chow (Columbia University), Kai Lai Chung (Stanford University), Samuel Karlin (University of Stanford), Rolf-Dieter Reiss, Sheldon Ross (University of Berkeley), Henry Teicher (Rutgers University) and others", Polansky, A.M. (2016).

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